THE BRITISH RUBBER PRODUCERS' RESEARCH ASSOCIATION

Publication No. 163

APPARATUS FOR THE MEASUREMENT OF THE DYNAMIC SHEAR MODULUS AND HYSTERESIS OF RUBBER AT LOW FREQUENCIES

by

W. P. FLETCHER and A. N. GENT

Reprinted from the *Journal of Scientific Instruments*, Vol. 29, pp. 186–188, June 1952

Apparatus for the measurement of the dynamic shear modulus and hysteresis of rubber at low frequencies

By W. P. Fletcher, B.Sc., A.Inst.P., and A. N. Gent, B.Sc., Grad.Inst.P., British Rubber Producers' Research Association, Welwyn Garden City

[*Paper received* 26 *November*, 1951]

An apparatus is described which subjects a rubber test-piece to a force in simple shear, varying sinusoidally with time in the frequency range 0·0017–17 c/s; the instantaneous values of force and displacement being measured by photoelectric pickups. From the display on the screen of a cathode-ray tube of the mechanical hysteresis loop described by the vibrating rubber, measurements are made which allow calculation of the dynamic shear modulus and hysteresis. Typical results are given.

INTRODUCTION

The mechanical hysteresis of rubber has been investigated in a number of different ways. A review of most of the literature on this subject prior to 1946 has been made by Dillon and Gehman(1) and these authors classify hysteresis measurement methods as follows:

(*a*) Low speed stress-strain loop.
(*b*) Impact resilience.
(*c*) Free vibration.
(*d*) Forced vibration at resonance.
(*e*) Forced vibration: non-resonance.

The apparatus which is to be described is of the last type and has some points of resemblance both to the well-known Roelig(2) machine and also to that described by Kornfel'd and Posnjak.(3) The principle of the method is the subjection of a rubber test piece to a force in simple shear, varying sinusoidally with time, the measurement of instantaneous values of force and displacement by photoelectric pickups and the display on the screen of a cathode-ray tube of the mechanical hysteresis loop described by the vibrating rubber.

DESCRIPTION OF APPARATUS

Details of the apparatus are shown in Fig. 1. An eccentric *A* of adjustable throw is driven from a 1 h.p. electric motor through a variable speed gear box giving output speeds of approximately 1 000–0·1 r.p.m. in eight logarithmic steps. By means of a connecting rod *B* the cam produces a reciprocating motion in a cross head *C*. This motion is approximately sinusoidal; in the least favourable case with the eccentric adjusted to its maximum throw it can be shown (see Appendix) that the departure from sinusoidal motion is nowhere more than 4%. The reciprocating motion is transmitted by a rigid rod to the upper segment of an elliptic spring *D*, the lower segment of which is rigidly coupled to a carriage *E*. The rubber test piece consists of two rubber cylinders *F* bonded to either end of a cylindrical steel block and this central block is locked securely into the carriage. The outer ends of the rubber cylinders are bonded to steel end plates which are clamped securely to the base casting of the apparatus. The elliptic spring has a linear load/deformation characteristic and its compliance in a vertical sense is at least 100 times that of the rubber test piece and thus with the machine in motion, the sinusoidal motion of the upper segment of the spring gives rise to a sinusoidal force on the centre block of the rubber test piece.

In order to carry out measurements over a wide range of temperatures, a sheet metal tank (not shown in Fig. 1) is attached to the base of the machine so that the test piece and carriage are completely submerged in a heating or cooling liquid contained in the tank. Using a mixture of alcohol and solid carbon dioxide a temperature of − 60° C may readily be attained. A careful check shows that over the whole range of frequency, amplitude and temperature employed the viscosity of the containing liquid has a negligible effect on the behaviour of the vibrating system.

The deformations of the spring and of the rubber are translated into electrical potentials by a photoelectric system shown diagramatically in Fig. 2. Light from a 6 V 108 W projector lamp *H* passes through a condenser lens *J* and a limiting aperture *K* before striking a semi-reflecting plate *L*. This plate splits the beam, one component passing through a system of prisms to produce uniform illumination of a horizontal slit S_1. The other component after reflexion illuminates uniformly a vertical slit S_2. The whole of the optical system so far described, excluding the slits S_1 and S_2, is enclosed in a light proof aluminium casing from which the two beams emerge through a plate glass window. Each slit is carried at the end of a tube containing focusing lenses and a photoelectric cell. The light flux falling upon either photocell is proportional to the area of the corresponding slit. The effective area of S_1 is varied by a shutter attached to the centre of the lower segment of the elliptic spring and hence

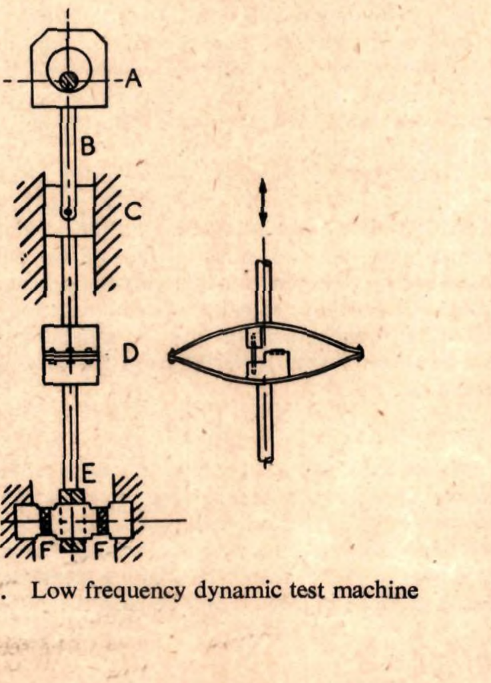

Fig. 1. Low frequency dynamic test machine

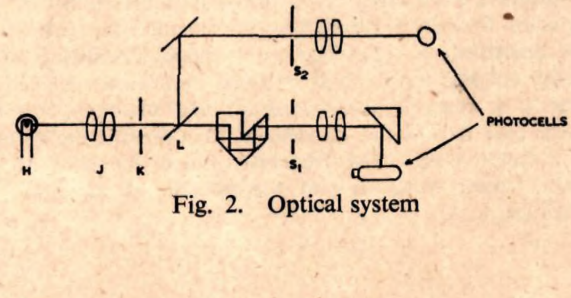

Fig. 2. Optical system

having the same vertical motion as the rubber carriage, while the effective area of slit S_2 is varied by the motion of two shutters, one attached to the upper and one to the lower segment of the spring so that any change in the spring deformation produces a proportionate change in the light flux through S_2.

The photoelectric cells are connected through the circuits shown in Fig. 3 to a cathode-ray oscilloscope having d.c. amplification on both X and Y axes. By means of the controls R_4 and R_6 the d.c. component voltages from the outputs of these coupling circuits are backed off, so that with the two shutters in their mean positions the oscillograph spot is in the centre of the screen.

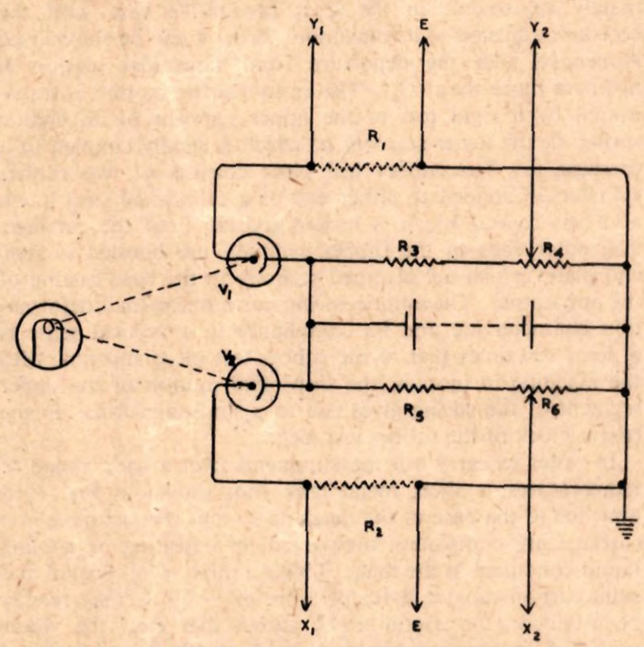

Fig. 3. Photocells and oscilloscope coupling circuit

R_1, R_2 = 1·2 MΩ (input resistance of oscilloscope)
R_3 = 5·9 MΩ
R_4 = 50 kΩ
R_5 = 1 MΩ
R_6 = 50 kΩ
V_1, V_2 = 90 AV
Y_1, Y_2, X_1, X_2, E = connexions to oscilloscope

The projector lamp is supplied with a current of 18 A at 6 V from a motor generator set with a heavy duty 6 V battery "floating" in the circuit. It is found desirable when carrying out precise measurements, to run from batteries alone since the slight ripple superimposed on the d.c. output of the generator causes a small degree of interference with the observed hysteresis pattern.

EVALUATION OF DYNAMIC PROPERTIES OF RUBBER

Results of dynamic mechanical tests have been presented in many different ways according to the purpose for which the results are required. In a previous publication(4) where a smaller frequency range was considered, the properties were described by reference to a model consisting of a parallel coupled spring and dashpot and were specified in terms of a dynamic shear modulus and viscosity. In the present case it is found convenient to represent the results, which cover a far wider frequency range, in terms of the dynamic shear modulus and the tangent of the angle of mechanical loss.

If an alternating force $y = F \cos \omega t$ (1)

be applied to the rubber test piece of stiffness S and viscous coefficient K, the following equation holds

$$K(dx/dt) + Sx = F \cos \omega t$$

where x is the instantaneous deformation. This has the solution

$$x = F \cos (\omega t - \delta)/(S^2 + \omega^2 K^2)^{\frac{1}{2}} \quad (2)$$

where $\delta = \tan^{-1} \omega K/S$

Equations (1) and (2) define the hysteresis loop, Fig. 4. When $x = 0$, $\omega t - \delta = (2n + 1)\pi/2$ and $y = F \cos \omega t = \pm F \sin \delta = \pm y_1$. The maximum value of $x = \pm F/(S^2 + \omega^2 K^2)^{\frac{1}{2}} = \pm x_2$. This is given by $\omega t - \delta = n\pi$, then $y = F \cos \omega t = \pm F \cos \delta = \pm y_2$. Now $S = (S^2 + \omega^2 K^2)^{\frac{1}{2}} \cos \delta = y_2/x_2$ and $\tan \delta = y_1/y_2$.

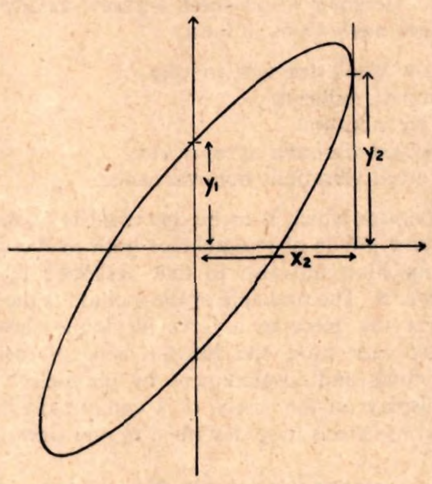

Fig. 4. The hysteresis loop

The hysteresis loop and a superimposed zero line along the x-axis are photographed on 35 mm recorder film and projected through an enlarger with about × 15 magnification on to graph paper ruled in mm. The elliptic spring having been calibrated by a dead load method the scales of the x and y oscilloscope deflexions are established by reading with a microscope the amplitudes of rubber and spring deformations and comparing these with the corresponding amplitudes of the projected oscillogram. Measurements on the projected hysteresis loop then allow the calculation of the absolute values of quantities x_2, y_1, and y_2.

An alternative method of measuring the angle of loss when this is high is to observe the steady wave forms of deflexion and force against time on the screen of the oscilloscope and measure directly the phase difference. The oscilloscope time base for this purpose is provided by a saw-tooth form voltage signal of the same period as that of the vibration, obtained by driving the moving contact of a full sweep potentiometer directly from the driving shaft of the eccentric.

RESULTS

As an example of the type of result obtained with the above apparatus, reference may be made to the dynamic properties of a Thiokol R.D. vulcanizate. The variation of dynamic shear modulus with frequency at several temperatures is shown in Fig. 5, the corresponding measurements of tan δ being given in Fig. 6. It will be seen that the tan δ versus ν

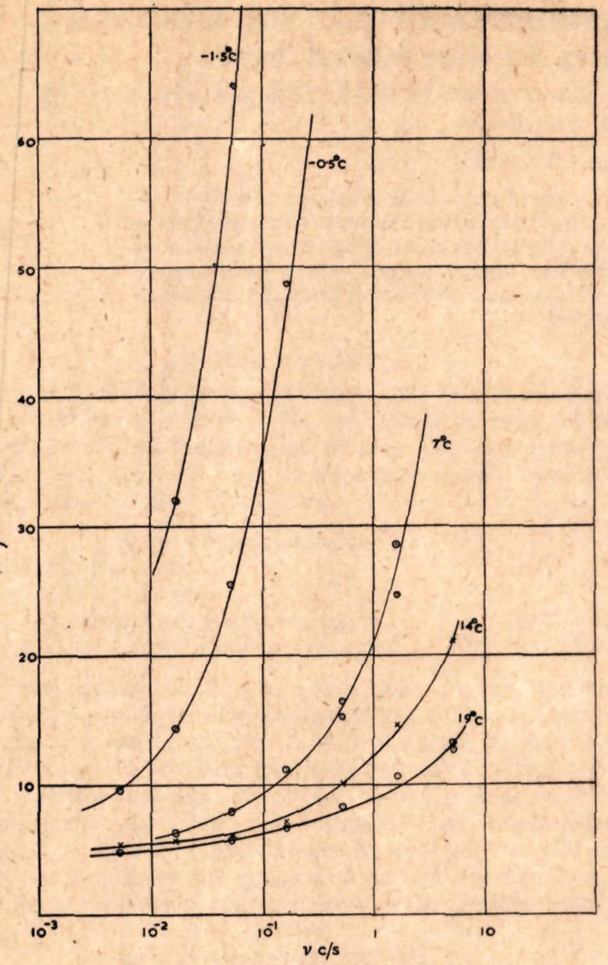

Fig. 5. Variation of dynamic shear modulus with frequency

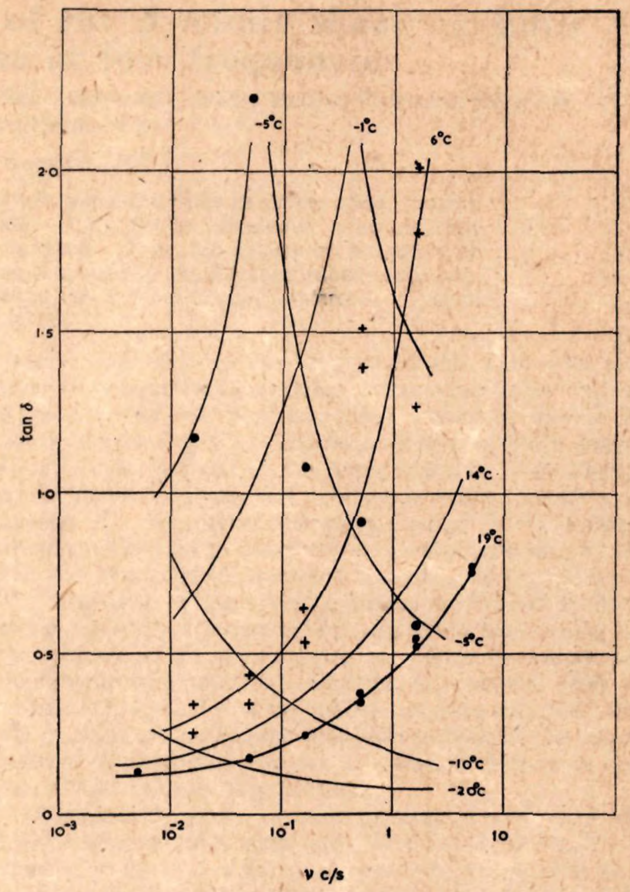

Fig. 6. Variation of mechanical loss angle with frequency

relationship at $-5°$ C and $-1°$ C passes through a sharp maximum. At other temperatures a single branch only of each curve is found within the frequency range employed. In order to avoid confusion experimental points are not indicated on all curves, but the scatter of the points indicates an increasing experimental error in regions of high tan δ. These results are in general agreement with data previously published for this polymer.[5, 6] The apparatus is in use for the investigation of the pattern of changes in dynamic properties of a range of natural and synthetic rubber-like polymers over a wide range of temperature and frequency. The full results of these measurements will be published elsewhere.

ACKNOWLEDGEMENT

Acknowledgement is made to Mr. D. Wilson for assistance in the design of the mechanical parts and to Mr. C. D. Kinloch for the design of the electronic section. This work forms part of a programme of technological research undertaken by the Board of the British Rubber Producers Research Association.

REFERENCES

(1) DILLON, J. H., and GEHMAN, S. D. *India Rubber World*, **115**, p. 61 (1946).

(2) ROELIG, H. *Proc. Rub. Tech. Conf., Lond.*, p. 821 (1938).

(3) KORNFEL'D, M., and POSNJAK, V. *J. Tech. Phys., USSR*, **9**, p. 4 (1939); *Rub. Chem. Tech.*, **13**, p. 136 (1940).

(4) FLETCHER, W. P., and GENT, A. N. *Trans Inst. Rubb. Ind.*, **26**, p. 45 (1950).

(5) MULLINS, L. *Trans Inst. Rubb. Ind.*, **22**, p. 235 (1947).

(6) FLETCHER, W. P., and SCHOFIELD, J. R. *J. Sci. Instrum.*, **21**, p. 193 (1944).

APPENDIX

Departure of the cross-head motion from a true sinusoidal form. For a crank radius r, and a connecting rod length l, we have

$$x/2r = (1 - \cos\theta)/2 + l[1 - (1 - r^2 \sin^2\theta/l^2)^{\frac{1}{2}}]/2r$$

for the displacement of the cross-head x as a fraction of the total throw, at a crank angle θ. The departure D from the corresponding displacement in the case of a true sinusoidal motion, is given by:

$$D = x/2r - (1 - \cos\theta)2 = l[1 - (1 - r^2 \sin{}^2\theta/l^2)^{\frac{1}{2}}]/2r \quad (1)$$

and is seen, on differentiating the right-hand side of equation (1), with respect to θ, to be a maximum when θ is 90°.

Substituting this value of θ, D is seen to be a monotonically decreasing function of l/r for values of l/r greater than 1. Using the minimum value of the ratio l/r used in practice, approximately 8, we obtain: $D_{max} = 0{\cdot}0313$.

The departure of the motion from a pure sinusoidal form having the same amplitude is seen never to exceed 4% of the total throw.

UNWIN BROTHERS LIMITED, WOKING AND LONDON