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NOTE

The Determination of the Bulk Modulus in a Constrained Solid

There appears at present to be some confusion concerning the measurement of the bulk modulus of polymers by means of uniaxial compression.

Matsuoka and Maxwell[1] used an apparatus of the cylinder and piston type to apply hydrostatic pressure to polymers in order to observe changes of state or phase under pressure and over a range of temperature. These workers were not concerned to measure accurate bulk moduli and Marvin and McKinney[2] point out that the method does not give a true bulk modulus due to the fact that all the deformation is uniaxial. Warfield et al.[3-7] used a similar apparatus to measure elastic constants for a number of polymers, and Warfield and Barnet[7] claim that the method gives values of bulk modulus agreeing well with values measured by other means. Nunziato, Schuler, and Walsh,[8] on the other hand, state that the method does not measure a true bulk modulus.

The relationship of the modulus as normally calculated using this method (apparent bulk modulus = B_a) to the true bulk modulus (B) may be obtained in a manner analogous to the conventional treatment of the relation between elastic moduli.[12]

Consider a cube of edge length l under uniaxial compressive stress σ. Let the change in length of the cube along the axis of stress be designated Δl. The change in the length of cube on sides normal to the stress axis will then be $\mu\Delta l$, where μ is Poisson's ratio. Then apply a hydrostatic pressure P sufficient to restore the lengths of the cube edges normal to the uniaxial stress axis to their original length l. This hydrostatic pressure is given by:

$$P = B\frac{\Delta V}{V} = \frac{3Bl^2\mu\Delta l}{(l + \Delta l)^2 (l - \Delta l)} \simeq \frac{3B\mu\Delta l}{l}$$

where Δl is taken as $\ll l$.

The apparent bulk modulus as normally calculated from the uniaxial distortion is then

$$B_a = \frac{(P + \sigma)l}{\Delta l(1 + \mu)l} = \frac{3B\mu + E}{1 + \mu}$$

where E is Young's modulus which is related to B and μ by the relation

$$E = 3B(1 - 2\mu) \tag{1}$$

which, when substituted in the previous equation, yields,

$$\frac{B_a}{B} = \frac{3(1 - \mu)}{1 + \mu} \tag{2}$$

It will be seen that according to this analysis, B_a is always greater than B over the range of μ encountered in practice, and that the higher the value of μ the smaller the error in the apparent modulus. The same result as equation (2) may be obtained directly from a tensor treatment.

By combining eqs. (1) and (2), it is seen that a value of μ may be obtained from the apparent modulus and Young's modulus using the quadratic equation.

$$2B_a\mu^2 + (B_a - E)\mu - (B_a - E) = 0 \tag{3}$$

A comparison of values of bulk modulus and Poisson's ratio for polystyrene obtained from the results of a number of workers is given in Table I. Where a uniaxial compression method has been used the values of apparent bulk modulus (B_a) and apparent Poisson's ratio (μ_a) obtained directly are given, and also the values B and μ obtained from

TABLE I
Values of Bulk Modulus and Poisson's Ratio for Polystyrene

$B_a \times 10^{-3}$ MNm^{-2}	$B \times 10^{-3}$ MNm^{-2}	$E \times 10^{-3}$ MNm^{-2}	μ_a	μ	Method	Ref.
5.4	3.60	3.6	0.39	0.333	uniaxial compression	5
4.4	2.63	3.4	0.37	0.295	uniaxial compression	6 6
—	3.81	—	—	0.344	acoustic	10
—	3.92	—	—	—	acoustic	11
—	3.33[a]	—	—	—	acoustic	13
—	3.66	—	—	—	acoustic	
—	—	—	—	0.33–0.36	extensometer	15

[a] Value obtained by extrapolation of results to zero pressure and 25°C.

the same results using eq. (3) to calculate μ and eq. (2) to calculate B from the value of μ so calculated.

Although, as Warfield et al.[6] point out, the use of data from measurements on samples from differing sources raises some difficulties, it is clear that the values of bulk modulus (B_a) obtained by the method of Matsuoka and Maxwell are high in comparison with those of the other workers quoted who in most cases used acoustic methods which would generally be expected to give *higher* results than a static measurement, and the same applies to the values of Poisson's ratio.

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