

## CALIBRATION AND ASSESSMENT OF SOIL TESTS FOR ESTIMATING FERTILIZER REQUIREMENTS

### II.\* FERTILIZER REQUIREMENTS AND AN EVALUATION OF SOIL TESTING

By J. D. COLWELL†

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#### Summary

The estimation of the phosphorus and potassium fertilizer requirements of wheat and potatoes from soil test calibration equations is described. The procedure is based on the estimation of yield response to fertilizer at the site represented by the soil test and the application of economic principles to the function thus established.

Soil testing may be justified on the basis of statistical tests of significance on these estimates. Since, however, testing can be expensive it should also be justified on economic grounds, such as by comparison of the profit obtained from the use of soil test estimates of fertilizer requirement with the profit from the use of some alternative estimate of fertilizer requirement. A preliminary evaluation may be made from a comparison of profits from soil test and average regional estimates of fertilizer requirement, using the calibration equation for the estimation of profits at different soil test levels. This method of evaluation separates the benefit due to testing from that due to the application of fertilizers. It is shown by this procedure that a soil test for the phosphorus fertilizing of wheat and for potassium fertilizing of potatoes in southern Ontario is worth while, given sufficient diversity of nutrient level within the region.

#### I. INTRODUCTION

The relationship between crop yield and fertilizer application rate can be represented as a polynomial function of fertilizer rate, and this function can in turn be represented as a polynomial function of soil analysis or soil test for the fertilizer nutrient. The derivation of these functions has been described in Part I of this series (Colwell 1967) and the present paper is concerned with their application for the estimation of optimal fertilizer requirements, as might be employed in a soil testing service to farmers. Such an application can be justified by the statistical tests of significance on the regression relationships between the soil test and yield response data. It should also be justifiable on the economic basis that the value of these regression estimates of fertilizer requirements is sufficiently better than alternative estimates, such as an average regional recommendation, to cover the cost of the soil test. This aspect of soil testing is also examined.

The soil test calibrations to be described were derived from data on the yield response of wheat and potatoes to nitrogen, phosphorus, and potassium fertilizers in Ontario, Canada. Soil test measurements for these respective nutrients were made

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† Division of Soils, CSIRO, P.O. Box 109, Canberra City, A.C.T. 2601.

on the surface soil of the sites of field experiments. Significant ( $P < 0.05$  to  $P < 0.001$ ) relationships were established between the phosphorus and potassium tests and some of the orthogonal trends of the fertilizer experiments, but no significant relationships were found for the nitrogen tests.

## II. MATHEMATICAL PROCEDURES

### (a) Calibration Equation

The calibration equations described in Part I are of the form

$$Y = P_0 \xi_0 + P_1 \xi_1 + P_2 \xi_2 + P_3 \xi_3, \quad (1)$$

where  $Y$  is the yield,  $P_0, \dots, P_3$  are coefficients, and  $\xi_0, \dots, \xi_3$  are orthogonal polynomials of fertilizer application rate, the subscripts denoting the order of the respective polynomials: 0 for nil trend (i.e. level of yield), 1 for linear trend, 2 for quadratic trend, and 3 for cubic trend, the cubic one being the highest order since fertilizer was only applied at four constant rates over all sites. Orthogonal polynomials in both the natural and square-root scales were used in Part I and it was shown that there was some advantage in the adoption of the square-root scale. Regressions of the form (1) were fitted to the data of each site and the coefficients for each site were then used as dependent variables to solve simultaneous regressions of the form

$$P_k = q_k + r_k T^{\frac{1}{2}} + s_k T, \quad k = 0, 1, 2, 3, \quad (2)$$

where  $T$  is the soil test measurement of sites and  $k$  corresponds to the order of the polynomials in (1). The coefficients  $q, r$ , and  $s$  may be regarded as regional parameters that provide a generalization of (1) by a function relating yield to fertilizer rate and soil test value. With the appropriate substitutions from (2) for the coefficients  $P_k$  in (1) and the expansion of the orthogonal polynomials as described below, a generalized function may be obtained in the form of polynomials of test  $T$  and fertilizer rate  $X$ , in the square-root scale, namely

$$Y = (\alpha_0 + \alpha_1 T^{\frac{1}{2}} + \alpha_2 T) + (\beta_0 + \beta_1 T^{\frac{1}{2}} + \beta_2 T)X^{\frac{1}{2}} + (\gamma_0 + \gamma_1 T^{\frac{1}{2}} + \gamma_2 T)X \\ + (\delta_0 + \delta_1 T^{\frac{1}{2}} + \delta_2 T)X^{3/2}. \quad (3)$$

Alternatively an average regional yield function

$$Y = a + bX^{\frac{1}{2}} + cX + dX^{3/2} \quad (4)$$

may be obtained without soil test regressions by averaging the coefficients of (1) over sites and again expanding and collecting terms. In both cases the coefficients may be regarded as regional parameters of generalized yield functions. Alternative to the derivation of (3) or (4) from (1) and (2), equations (3) or (4) may be estimated directly by a least square fit of the regressions to the yield, fertilizer, and soil test data using models (3) or (4). It can be shown that the coefficients obtained by this direct procedure are identical with those derived from equations (1) and (2) for the same data. This latter procedure does not, however, allow valid significance testing, as discussed in Part I.

The polynomial expressions of soil test  $T$  in equations (2) and (3) are also in the square-root scale. This scale was chosen here also because it was found to give a somewhat better fit of the data for model (2) compared with the corresponding quadratic expressions in the natural scale.

TABLE 1  
REGRESSION COEFFICIENTS FOR EQUATIONS (2) AND VARIANCE RATIO FOR  
REDUCTION IN SUM OF SQUARES DUE TO REGRESSION

The regressions are for orthogonal coefficients of yield response to fertilizers by wheat and potatoes as functions of nitrogen, phosphorus, and potassium soil tests

Polynomial Coefficients	Regression Coefficients			Variance Ratio	Regression Coefficients			Variance Ratio
	$q$	$r$	$s$		$q$	$r$	$s$	
Wheat-Nitrogen					Potatoes-Nitrogen			
$P_0$	34.23	-0.1940	7.457	1.37	533.8	-2.21	46.30	0.08
$P_1$	-40.32	-0.4518	9.101	2.61	405.7	3.74	-74.66	2.30
$P_2$	-9.18	-0.0361	1.241	0.94	-30.3	-0.68	9.21	0.28
$P_3$	-0.89	-0.0579	0.618	2.01	26.1	0.54	-7.77	0.73
Wheat-Phosphorus					Potatoes-Phosphorus			
$P_0$	20.48	-0.1984	8.000	4.28*	-169.5	-1.74	83.76	0.96
$P_1$	19.78	0.0338	-1.704	6.65**	-78.7	-0.42	17.45	0.39
$P_2$	-0.89	0.0271	-0.547	3.26	-14.2	0.22	-5.60	7.52**
$P_3$	-5.03	-0.0233	0.696	0.38	-31.5	-0.08	3.20	0.11
Wheat-Potassium					Potatoes-Potassium			
$P_0$	70.59	-0.0205	1.659	0.38	690.8	0.04	4.18	0.05
$P_1$	23.17	0.0981	-3.116	3.94*	1177.5	5.29	-155.95	16.29***
$P_2$	-9.14	-0.0296	0.982	0.57	262.1	2.19	-50.78	2.78
$P_3$	-8.98	-0.0605	1.475	2.91	-116.9	-0.75	18.71	0.59

TABLE 2  
VALUES OF SQUARE ROOT OF FERTILIZER RATE  $Z$  AND OF CONSTANTS  
FOR ORTHOGONAL POLYNOMIALS IN EQUATION (5)

Z and Constants	Wheat			Potatoes		
	Nitrogen	Phosphorus	Potassium	Nitrogen	Phosphorus	Potassium
$Z_1$	$\sqrt{0}$	$\sqrt{0}$	$\sqrt{0}$	$\sqrt{0}$	$\sqrt{0}$	$\sqrt{0}$
$Z_2$	$\sqrt{12.5}$	$\sqrt{25}$	$\sqrt{25}$	$\sqrt{25}$	$\sqrt{50}$	$\sqrt{50}$
$Z_3$	$\sqrt{25}$	$\sqrt{50}$	$\sqrt{50}$	$\sqrt{50}$	$\sqrt{100}$	$\sqrt{100}$
$Z_4$	$\sqrt{50}$	$\sqrt{120}$	$\sqrt{120}$	$\sqrt{100}$	$\sqrt{200}$	$\sqrt{200}$
$\bar{Z}$	3.90	5.76	5.76	5.52	7.80	7.80
$a_0$	0.50	0.50	0.50	0.50	0.50	0.50
$b_1$	0.194	0.127	0.127	0.137	0.097	0.087
$a_2$	-0.581	-0.546	-0.546	-0.581	-0.581	-0.581
$b_2$	0.0863	0.0271	0.0271	0.0610	0.0431	0.0431
$c_2$	0.0873	0.0350	0.0350	0.0437	0.0218	0.0218
$a_3$	0.231	0.135	0.135	0.231	0.231	0.231
$b_3$	-1.055	-0.707	-0.707	-0.746	-0.528	-0.528
$c_3$	0.0524	0.0103	0.0103	0.0262	0.0131	0.0131
$d_3$	0.0881	0.0244	0.0244	0.0312	0.0110	0.0110

The equation (3) is obtained from (1) and (2) by expanding the orthogonal polynomials in (1)

$$\left. \begin{aligned} \xi_0 &= a_0, \\ \xi_1 &= b_1(Z - \bar{Z}), \\ \xi_2 &= a_2 + b_2(Z - \bar{Z}) + c_2(Z - \bar{Z})^2, \\ \xi_3 &= a_3 + b_3(Z - \bar{Z}) + c_3(Z - \bar{Z})^2 + d_3(Z - \bar{Z})^3 \end{aligned} \right\} \quad (5)$$

and substituting the regressions  $P_k$  from (2). Here  $Z = X^{\frac{1}{2}}$  (the square root of the fertilizer rate) to give polynomials in the square-root scale, and  $a_0, b_1, \dots, d_3$  are constants chosen to secure the condition of orthogonality. Regression estimates of the parameters  $q_k, r_k$ , and  $s_k$  in (2) and values for the constants  $a_0, b_1, \dots, d_3$  in (5) appropriate to the present calibrations are listed in Tables 1 and 2. A form of expansion corresponding to (3) or (4) and convenient for the computation of yield is

$$\begin{aligned} Y &= \{P_0 a_0 - P_1 b_1 + P_2(a_2 - b_2 \bar{Z} + c_2 \bar{Z}^2) + P_3(a_3 - b_3 \bar{Z} + c_3 \bar{Z}^2 - d_3 \bar{Z}^3)\} \\ &\quad + \{(P_1 + P_2(b_2 - 2c_2 \bar{Z}) + P_3(b_3 - 2c_3 \bar{Z} + 3d_3 \bar{Z}^2))X^{\frac{1}{2}} \\ &\quad + \{P_2 c_2 + P_3(c_3 - 3d_3 \bar{Z})\}X + \{P_3 d_3\}X^{3/2}, \end{aligned} \quad (6)$$

where the  $P_0, \dots, P_3$  are either functions of soil test as defined by the regressions (2), to give a function corresponding to (3), or the means of site coefficients, to give an average regional function corresponding to (4).

#### (b) Fertilizer Requirement

The economic principles that apply to fertilizer use and to the interpretation of fertilizer experiments have been described by, for example, Heady, Pesek, and Brown (1955), Munson and Doll (1959), Heady and Dillon (1960), Heady (1961), and Colwell and Esdaile (1966). Thus if net profit from the application of fertilizer is defined as

$$\pi_j = \Delta Y_j V - I_j, \quad (7)$$

where  $\pi_j$  is the profit for a fertilizer rate  $j$ ,  $V$  is the value of the crop, and  $I_j = K X_j$  is the investment in fertilizer, with  $X_j$  the rate of application and  $K$  the cost per unit of applied fertilizer, then the rate of return on a differential of investment is the slope

$$R = \partial \pi / \partial I. \quad (8)$$

Since the rate of return  $R$  decreases as  $I$  increases, a minimal value of  $R$  may be defined by economic considerations of alternative investments available to the farmer and of risks inherent in cropping, etc., and this limiting value of  $R$  may be used to calculate an optimal value of  $I$  or an economic optimal fertilizer rate. Thus fertilizer requirement can be defined as the amount of fertilizer required to secure response up to a limiting value of  $R$  and can be calculated by appropriate solution of equation (8).

Yield response, profit, and fertilizer requirement can be calculated by appropriate substitutions from equations (3) or (4). For the calibration equations, however, it



was found most convenient to use equation (6) omitting the first bracketed expression to obtain  $\Delta Y$  the yield response. Thus for profit

$$\pi = [\{P_1 b_1 + P_2(b_2 - 2c_2 \bar{Z}) + P_3(b_3 - 2c_3 \bar{Z} + 3d_3 \bar{Z}^2)\}X^{\frac{1}{2}} + \{P_2 c_2 + P_3(c_3 - 3d_3 \bar{Z})\}X + \{P_3 d_3\}X^{3/2}]V - KX, \quad (9)$$

and fertilizer requirement can be calculated by solution of the quadratic that is obtained by differentiating (9) with respect to investment,  $I$  or  $KX$ , with appropriate values for  $K$ ,  $V$ , and  $R$ , namely

$$\frac{1}{2}\{P_1 b_1 + P_2(b_2 - 2c_2 \bar{Z}) + P_3(b_3 - 2c_3 \bar{Z} + 3d_3 \bar{Z}^2)\} + \{P_2 c_2 + P_3(c_3 - 3d_3 \bar{Z}) - (K/V)(R+1)\}X^{\frac{1}{2}} + \frac{3}{2}\{P_3 d_3\}X = 0. \quad (10)$$

Since fertilizer requirement as defined by (8) or (10) depends on yield response rather than yield, only the coefficients  $P_1$ ,  $P_2$ , and  $P_3$  need be estimated. Where these coefficients are estimated from the regressions (2) with appropriate substitution of soil test  $T$ , the estimate of fertilizer requirement is the soil test estimate. Where the coefficients are, on the other hand, means of coefficients in (1) for the region, the estimate is of the average regional fertilizer requirement.

### III. RESULTS AND DISCUSSION

The calibration equations provide estimates of yield response, profit functions, or economic fertilizer requirements given appropriate values for  $V$ , the value of the crop,  $K$ , the cost of fertilizer, and  $R$ , the minimal marginal rate of return on the investment in fertilizer. Appropriate values of  $V$ ,  $K$ , and  $R$  should be estimated from local economic studies. For the present study, values, costs, and marginal rates of return have been assumed on the basis of local experience in Ontario in 1966. These will vary from site to site and with time. Thus wheat has been valued at \$1.75 (Canadian) per bushel, potatoes at \$2.00 per cwt (100 lb), and fertilizers at 12c per lb N, 10c per lb  $P_2O_5$ , and 5c per lb  $K_2O$ . Optimal marginal rate of return on investment in fertilizer has been chosen arbitrarily at  $R = 25\%$  for the present calculations. Some alternative values of  $R$  ranging from 0 to 100% have also been used in the illustrations, thus covering the range of appropriate values likely to be established by economic studies.

#### (a) Generalized Yield Response Functions

The validity of the soil tests as guides to fertilizer requirements has been judged on the basis of the significance of the regressions on the coefficients  $P_1$ ,  $P_2$ , and  $P_3$  of the orthogonal polynomials (Table 1). The soil tests are, however, proportionately more effective than suggested by these tests of significance, since an appreciable part of the residual variance would be due to experimental error associated with the data from the field experiments. The selection of the probability level  $P = 0.05$  for significance, is of course arbitrary. Where a trend is expected from experience with significant regressions on similar data and where a trend is in fact indicated by a regression variance ratio greater than one, nonsignificant trends may be used on the

basis that they provide best estimates. The phosphorus and potassium tests gave significant ( $P < 0.05$ ) regressions with some of the coefficients for the first- and second-order polynomial coefficients and in these instances the nonsignificant trends on the other coefficients may be used for the estimation of the remaining first- and second-order coefficients. This applies particularly to the regression on  $P_2$  for the wheat-phosphorus data (Table 1), where the variance ratio of 3.26 on the reduction due to regression is only slightly less than the variance ratio level of 3.31 for significance at  $P = 0.05$ . Accordingly the regressions on  $P_1$  and  $P_2$  in Table 1 have been used to estimate these coefficients for the estimation of phosphorus and potassium fertilizer requirements in the present calibrations. No significant regressions were established for the  $P_3$  coefficient with any of the data. Consequently the mean  $P_3$  value has been taken as the best estimate of this coefficient for the generalized function, although in practice the use of the regressions to estimate  $P_3$  in these instances makes little difference to the estimate of yield response.

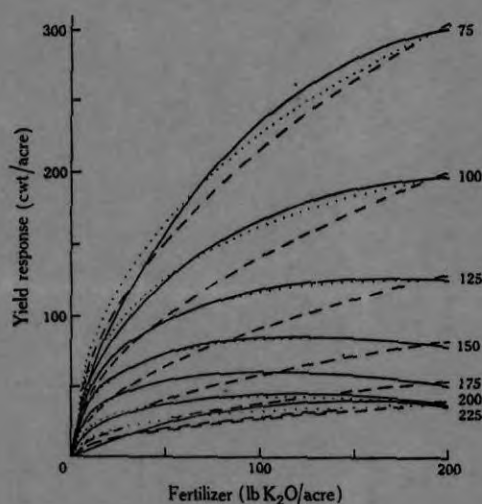


Fig. 1.—Yield response to fertilizer at the indicated soil test levels (in parts per two million) for the potatoes-potassium test as predicted by the calibration equations:

- (11),  $\xi_1$   
 ..... (12),  $\xi_1, \xi_2$   
 — (13),  $\xi_1, \xi_2, \xi_3$

The nitrogen test gave, however, only nonsignificant ( $P > 0.05$ ) regressions on the coefficients  $P_1$ ,  $P_2$ , or  $P_3$  of the orthogonal polynomials (Table 1), so that the best generalized yield response function provided by the present data is the mean yield response obtained by averaging over site values of  $P_1$ ,  $P_2$ , and  $P_3$ .

In general the greater part of the yield response information is provided by the lower order polynomials and, as mentioned previously, it was to secure this effect that the square-root scale was selected for the orthogonal polynomials. The relative contribution of the polynomials to the generalized response function is illustrated in Figure 1, which graphs, for a range of soil test values, the predicted yield response obtained from the equations

$$\Delta Y = P_1 \xi_1, \quad (11)$$

$$\Delta Y = P_1 \xi_1 + P_2 \xi_2, \quad (12)$$

and

$$\Delta Y = P_1 \xi_1 + P_2 \xi_2 + P_3 \xi_3 \quad (13)$$

for the soil test calibrations on the potatoes-potassium data. The coefficients  $P_1$  and  $P_2$  have been estimated for this illustration from the regression equations and  $P_3$  from the mean of site coefficients for the cubic trend. The regression on  $P_1$  is the most highly significant and this regression (dashed lines in Fig. 1) accounts for most of the trend in yield response as a function of soil test. The contribution of the second-order polynomial coefficient, equation (12), is shown by the dotted lines in Figure 1. The effect is substantial and reflects the overall magnitude and importance of the second-order term. The effect of the relatively low order of significance of the regression on  $P_2$  is shown by the relatively small variation in this contribution for different soil test values, as can be seen by comparison of the dashed and dotted lines in Figure 1, for different soil test values.

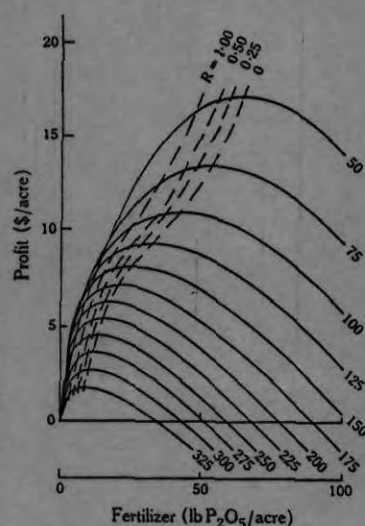


Fig. 2.—Profit from phosphorus fertilizer applied to wheat at the indicated soil test levels (in parts per two million). The dashed lines are the isoclines for  $R = 0, 0.25, 0.50$ , and  $1.00$  on the profit  $\times$  fertilizer  $\times$  soil test surface satisfying equation (10).

The addition of the cubic trend (equation (13) and full lines in Fig. 1) has only a small effect, corresponding to a relatively small value for the mean of the coefficients  $P_3$ .

Similar effects from the addition of the orthogonal terms, corresponding to those in Figure 1, were observed for the other calibrations.

#### (b) Profit and Fertilizer Requirement

Yield response to fertilizer is of interest to farmers because of the profit thereby implied. The generalized yield response function, as illustrated in Figure 1, may be represented alternatively as a generalized profit function by equation (9), and this is illustrated for the wheat-phosphorus test calibration in Figure 2. Such predicted profit functions show the increase in profit and decrease in slope with increase in fertilizer application up to a point of maximum profit, typical of the law of diminishing returns. The intersecting set of curves (dashed lines) in Figure 2 are lines of constant slope, or isoclines, on the profit versus fertilizer versus soil test surface. They represent the trace of points satisfying equation (10) for fertilizer

requirement over a continuous range of soil test values with the indicated values of  $R$ . The points of intersection correspond to fertilizer requirements for the particular values of soil test  $T$  and slope  $R$ . For fertilizer applications greater than the point of maximum profit at  $R = 0$ , the slopes of the profit function become negative with respect to fertilizer rate, indicating a marginal loss on the investment in fertilizer. At the point of maximum profit the marginal rate of return is nil so that ordinarily fertilizer will not be applied up to this level. Rather a value of  $R > 0$ , as for  $R = 0.25$ ,  $0.50$ , and  $1.00$  in Figure 2, will be chosen to ensure a profit on the last differential of investment in fertilizer.

TABLE 3  
FERTILIZER REQUIREMENTS FOR MARGINAL RATE OF RETURN  $R = 0.25$   
AND CORRESPONDING PREDICTED PROFIT

Test	Fertilizer (lb/acre)	Profit (\$/acre)	Test	Fertilizer (lb/acre)	Profit (\$/acre)
Wheat-Nitrogen			Potatoes-Nitrogen		
—	27.8	5.12	—	132.8	122.07
Wheat-Phosphorus			Potatoes-Phosphorus		
50	60.0	17.05	—	—	—
100	37.0	10.85	50	52.0	115.60
150	22.0	8.02	100	72.3	172.46
200	15.3	6.21	200	96.9	228.48
250	11.6	4.48	300	117.9	249.20
300	8.7	2.68	400	145.1	252.20
Wheat-Potassium			Potatoes-Potassium		
75	50.6	8.40	50	242.3	932.89
100	31.1	4.52	75	229.29	598.91
125	15.8	2.65	100	203.15	382.22
150	9.2	1.58	150	113.21	164.30
175	5.4	0.67	200	106.09	85.39

Fertilizer requirement of potatoes and wheat for phosphorus and potassium, and the corresponding expected profits have been calculated for a range of soil test values using appropriate substitutions from Tables 1 and 2 in equations (9) and (10) and a marginal rate of return  $R = 0.25$  (Table 3). Some of the values can also be read directly from the curves in Figure 2 for the wheat-phosphorus calibrations. The fertilizer requirements are expected to decrease with increase in amount of the fertilizer nutrient in the soil, as represented by the soil test, and this is shown by the inverse relationships between soil test and fertilizer requirements in Table 3 for the wheat-phosphorus, wheat-potassium, and potatoes-potassium calibrations. A reverse trend is shown, however, by the potatoes-phosphorus calibration. An examination of the yield data used for this calibration shows that for high soil test values the yields for nil fertilizer are abnormally low so that the response to fertilizer is greater on these richer soils. This could be due to experimental error and points to a need for more field data on such sites. The calibration simply reproduces the abnormality that is sufficiently consistent to ensure a significant relationship between the test and the quadratic trend (Table 1). As noted in Part I, this particular cali-



bration accounted for only a small proportion of the yield response to phosphorus fertilizer. The potatoes-phosphorus calibration is thus suspect despite its statistical significance.

For the nitrogen test data no significant regressions were established to justify the prediction of fertilizer requirement or profit from test values. For these the only general yield response function provided by the present data thus becomes the average response function obtained by averaging the coefficients  $P_1$ ,  $P_2$ , and  $P_3$  over sites. An estimate of the average regional fertilizer requirement obtained from this function has therefore been calculated from equation (10) using means rather than soil test estimates of  $P_1$ ,  $P_2$ , and  $P_3$ . The corresponding average profit (9) thus obtained is also given in Table 3.

### *(c) Evaluation of Soil Testing*

A direct estimate of the value of testing to a region could be obtained ideally from fertilizer experiments that were distributed to represent the region and designed to measure the difference of net profit from the yield response to a test recommendation from that of some alternative recommendation. In the absence of such data a preliminary evaluation may be made from the calibration equations by comparing expected profit from soil test recommendations for particular soil test values and a chosen value of  $R$  (equation (10)) with that from an average recommendation. Assuming that the soil test generalization of the profit function provides an accurate estimate of profit from fertilizer for sites of particular soil test levels, the profit from the use of the average fertilizer requirement for particular test levels can be calculated by substitution in this generalization. Alternative regional fertilizer recommendations, such as those of established practice, might likewise be used as a basis to evaluate soil test recommendations.

Evaluations of soil testing for the phosphorus fertilizer requirements of wheat and the potassium fertilizer requirements of potatoes have been made using the respective soil test generalization of equation (9) illustrated in Figure 2,  $R = 0.25$ , and average fertilizer requirements of 22.6 lb  $P_2O_5$  and 3.9 lb  $K_2O$ , which were estimated from the average yield response of the field experiments. These two calibrations were chosen since they gave the most highly significant correlations in this study. The fertilizer requirement columns headed "test" in Table 4 have thus been estimated by substitution of  $P_1$  and  $P_2$  values estimated by the regressions for the particular soil test levels (Table 1) and the mean  $P_3$  in equation (10). The fertilizer values in the columns headed "average" have been estimated, on the other hand, by substitution of the respective mean values of  $P_1$ ,  $P_2$ , and  $P_3$  in equation (10). The profits corresponding to these alternative fertilizer applications have been calculated by equation (9), and the difference between these profits represents the gain from following the soil test recommendation over that from an average recommendation. This is an estimate of gross gain, ignoring the cost of the soil testing operation, and actual net gain would be less this cost.

The gross gains from the use of the soil test fertilizer recommendations (Table 4) are negligible when they are in the vicinity of the average recommendation, and only become appreciable with considerable divergence of the requirements on either side

of the average. This is an expression of the typically low curvature of the profit functions in the vicinity of the optimum, i.e. in the vicinity of the intersections in Figure 1. The valuation thus illustrates the need for the dissociation of the benefits to be derived from soil testing from those due to the application of fertilizer. These latter benefits can be much greater. Thus, in a district that is uniformly very deficient in a particular nutrient, the benefit from fertilizers will be high, but since the district is uniform the financial return from soil test recommendations will be almost identical with that from a proper estimate of the average fertilizer requirement, so that soil

TABLE 4  
ECONOMIC EVALUATION OF SOIL TESTING FOR PHOSPHORUS FERTILIZER  
REQUIREMENTS OF WHEAT AND POTASSIUM FERTILIZER REQUIREMENTS OF POTATOES  
Fertilizer requirements are estimated from the soil test calibrations, and the average response to fertilizer and the profits are calculated by substitution in equation (9)

Test	Fertilizer Requirement (lb/acre)		Predicted Profit (\$/acre)		Gain from Test (\$/acre)
	(Test)	(Average)	(Test)	(Average)	
Wheat-Phosphorus					
50	60.0	22.6	17.05	11.60	5.45
100	37.0	22.6	10.85	9.09	1.76
200	15.3	22.6	6.19	6.15	0.04
300	7.0	22.6	2.68	2.08	0.60
Potatoes-Potassium					
50	242.3	182.2	932.89	884.31	48.58
100	203.2	182.2	382.22	379.67	2.55
150	113.2	182.2	164.30	154.19	10.11
200	106.1	182.2	85.39	73.23	12.16

testing could cost more than the value of the greater precision it provides. Thus, on a regional basis, soil testing will be of most value where requirements cover a wide range, and a proper evaluation of the present tests would require some information on the heterogeneity of southern Ontario. The soil test will of course still be valuable to identify homogeneous districts within a region and to the farmer who wants a guide for progressively reducing fertilizer rates as the nutrient level of the soil is raised over a term of periodic applications. There is also the rather indeterminable value of soil testing as a vehicle for extension recommendations.

#### IV. ACKNOWLEDGMENTS

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